HL Paper 1

A point P, relative to an origin O, has position vector $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix}$, $s \in \mathbb{R}$.

Find the minimum length of $\overrightarrow{\mathrm{OP}}$.

Three distinct non-zero vectors are given by $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$, and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

If $\overrightarrow{\mathrm{OA}}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$ and $\overrightarrow{\mathrm{OB}}$ is perpendicular to $\overrightarrow{\mathrm{CA}}$, show that $\overrightarrow{\mathrm{OC}}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$.

Consider the vectors $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{a} + \boldsymbol{b}$. Show that if $|\boldsymbol{a}| = |\boldsymbol{b}|$ then $(\boldsymbol{a}+\boldsymbol{b})\cdot(\boldsymbol{a}-\boldsymbol{b}) = 0$. Comment on what this tells us about the parallelogram OACB.

Consider the plane with equation $4x - 2y - z = 1$ and the line given by the parametric equations

$x = 3 - 2\lambda$

$y = (2k-1) + \lambda$

$z = -1 + k\lambda.$

Given that the line is perpendicular to the plane, find

(a) the value of k;

(b) the coordinates of the point of intersection of the line and the plane.

Two boats, A and B , move so that at time t hours, their position vectors, in kilometres, are $\boldsymbol{r}_A = (9t)\boldsymbol{i} + (3-6t)\boldsymbol{j}$ and $\boldsymbol{r}_B = (7-4t)\boldsymbol{i} + (7t-6)\boldsymbol{j}$.

a. Find the coordinates of the common point of the paths of the two boats. [4]

b. Show that the boats do not collide. [2]

O, A, B and C are distinct points such that $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

It is given that ***c*** is perpendicular to $\overrightarrow{AB}$ and ***b*** is perpendicular to $\overrightarrow{AC}$.

Prove that ***a*** is perpendicular to $\overrightarrow{BC}$.

The points A, B, C and D have position vectors ***a***, ***b***, ***c*** and ***d***, relative to the origin O.

It is given that $\vec{AB} = \vec{DC}$.

The position vectors $\vec{OA}$, $\vec{OB}$, $\vec{OC}$ and $\vec{OD}$ are given by

$\boldsymbol{a} = \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k}$

$\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k}$

$\boldsymbol{c} = q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$

$\boldsymbol{d} = -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k}$

where *p* , *q* and *r* are constants.

The point where the diagonals of ABCD intersect is denoted by M.

The plane Π cuts the *x*, *y* and *z* axes at X , Y and Z respectively.

a.i. Explain why ABCD is a parallelogram. [1]

a.ii. Using vector algebra, show that $\vec{AD} = \vec{BC}$. [3]

b. Show that *p* = 1, *q* = 1 and *r* = 4. [5]

c. Find the area of the parallelogram ABCD. [4]

d. Find the vector equation of the straight line passing through M and normal to the plane Π containing ABCD. [4]

e. Find the Cartesian equation of Π. [3]

f.i. Find the coordinates of X, Y and Z. [2]

f.ii. Find YZ. [2]

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a. Show that the points $\mathrm{O}(0,\ 0,\ 0),\ \mathrm{A}(6,\ 0,\ 0), \mathrm{B}(6,\ -\sqrt{24},\ \sqrt{12}), \mathrm{C}(0,\ -\sqrt{24},\ \sqrt{12})$ form a square. [3]

b. Find the coordinates of M, the mid-point of [OB]. [1]

c. Show that an equation of the plane *Π*, containing the square OABC, is $y + \sqrt{2}z = 0$. [3]

d. Find a vector equation of the line L, through M, perpendicular to the plane *Π*. [3]

e. Find the coordinates of D, the point of intersection of the line L with the plane whose equation is $y = 0$. [3]

f. Find the coordinates of E, the reflection of the point D in the plane *Π*. [3]

g. (i) Find the angle $\mathrm{O\hat{D}A}$. [6]

(ii) State what this tells you about the solid OABCDE.

(a) Show that the two planes

$$\pi_1 : x + 2y - z = 1$$

$$\pi_2 : x + z = -2$$

are perpendicular.

(b) Find the equation of the plane π_3 that passes through the origin and is perpendicular to both π_1 and π_2.

The following system of equations represents three planes in space.

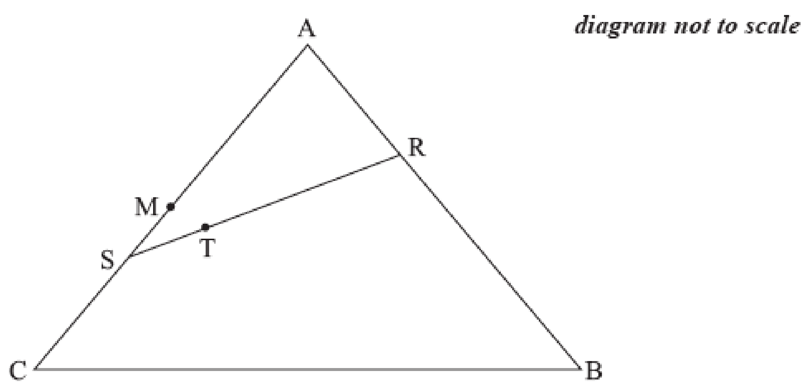
$$x + 3y + z = -1$$

$$x + 2y - 2z = 15$$

$$2x + y - z = 6$$

Find the coordinates of the point of intersection of the three planes.

The position vectors of the points A, B and C are a, b and c respectively, relative to an origin O. The following diagram shows the triangle ABC and points M, R, S and T.



M is the midpoint of $[AC]$.

R is a point on $[AB]$ such that $\overrightarrow{\mathrm{AR}} = \frac{1}{3}\overrightarrow{\mathrm{AB}}$.

S is a point on $[AC]$ such that $\overrightarrow{\mathrm{AS}} = \frac{2}{3}\overrightarrow{\mathrm{AC}}$.

T is a point on $[RS]$ such that $\overrightarrow{\mathrm{RT}} = \frac{2}{3}\overrightarrow{\mathrm{RS}}$.

a. (i) Express $\overrightarrow{\mathrm{AM}}$ in terms of a and c. [4]

(ii) Hence show that $\overrightarrow{\mathrm{BM}} = \frac{1}{2}a - b + \frac{1}{2}c$.

b. (i) Express $\overrightarrow{\mathrm{RA}}$ in terms of a and b. [5]

(ii) Show that $\overrightarrow{RT} = -\frac{2}{9}a - \frac{2}{9}b + \frac{4}{9}c$.

c. Prove that T lies on $[BM]$. [5]

Find the values of x for which the vectors $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix}$ are perpendicular, $0 \leqslant x \leqslant \frac{\pi}{2}$.

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 2\boldsymbol{k}$.

a. Find $\boldsymbol{a} \times \boldsymbol{b}$. [2]

b. Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and passing through the point $(1,\ 0,\ -1)$. [3]

The points A(1, 2, 1) , B(−3, 1, 4) , C(5, −1, 2) and D(5, 3, 7) are the vertices of a tetrahedron.

a. Find the vectors $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [2]

b. Find the Cartesian equation of the plane $\prod$ that contains the face ABC. [4]

Consider the points $\mathrm{A}(1, 0, 0)$, $\mathrm{B}(2, 2, 2)$ and $\mathrm{C}(0, 2, 1)$.

A third plane Π_3 is defined by the Cartesian equation $16x + \alpha y - 3z = \beta$.

a. Find the vector $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. [4]

b. Find an exact value for the area of the triangle ABC. [3]

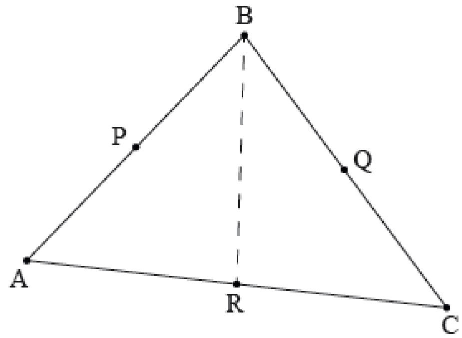
c. Show that the Cartesian equation of the plane Π_1, containing the triangle ABC, is $2x + 3y - 4z = 2$. [3]

d. A second plane Π_2 is defined by the Cartesian equation $\Pi_2 : 4x - y - z = 4$. L_1 is the line of intersection of the planes Π_1 and Π_2. [5]

Find a vector equation for L_1.

e. Find the value of α if all three planes contain L_1. [3]

f. Find conditions on α and β if the plane Π_3 does **not** intersect with L_1. [2]



Consider the triangle ABC. The points P, Q and R are the midpoints of the line segments $[AB]$, $[BC]$ and $[AC]$ respectively.

Let $\overrightarrow{\mathrm{OA}} = a$, $\overrightarrow{\mathrm{OB}} = b$ and $\overrightarrow{\mathrm{OC}} = c$.

a. Find $\overrightarrow{\mathrm{BR}}$ in terms of a, b and c. [2]

b. (i) Find a vector equation of the line that passes through B and R in terms of a, b and c and a parameter λ. [9]

(ii) Find a vector equation of the line that passes through A and Q in terms of a, b and c and a parameter μ.

(iii) Hence show that $\overrightarrow{\mathrm{OG}} = \frac{1}{3}(a + b + c)$ given that G is the point where $[BR]$ and $[AQ]$ intersect.

c. Show that the line segment $[CP]$ also includes the point G. [3]

d. The coordinates of the points A, B and C are $(1,\ 3,\ 1)$, $(3,\ 7,\ -5)$ and $(2,\ 2,\ 1)$ respectively. [9]

A point X is such that $[GX]$ is perpendicular to the plane ABC.

Given that the tetrahedron $ABCX$ has volume $12\ \mathrm{units}^3$, find possible coordinates of X.

Consider the lines l_1 and l_2 defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \text{ and } l_2 : \frac{6-x}{3} = \frac{y-2}{4} = 1 - z \text{ where } a \text{ is a constant.}$$

Given that the lines l_1 and l_2 intersect at a point P,

a. find the value of a; [4]

b. determine the coordinates of the point of intersection P. [2]

Consider the plane Π_1, parallel to both lines L_1 and L_2. Point C lies in the plane Π_1.

The line L_3 has vector equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}$.

The plane Π_2 has Cartesian equation $x + y = 12$.
The angle between the line L_3 and the plane Π_2 is 60°.

a. Given the points A(1, 0, 4), B(2, 3, −1) and C(0, 1, − 2) , find the vector equation of the line L_1 passing through the points A and B. [2]

b. The line L_2 has Cartesian equation $\frac{x-1}{3} = \frac{y+2}{1} = \frac{z-1}{-2}$. [5]

Show that L_1 and L_2 are skew lines.

c. Find the Cartesian equation of the plane Π_1. [4]

d. (i) Find the value of k. [7]

(ii) Find the point of intersection P of the line L_3 and the plane Π_2.

Given any two non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that $|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2$.

The points A and B are given by $\mathrm{A}(0,\ 3,\ -6)$ and $\mathrm{B}(6,\ -5,\ 11)$.

The plane Π is defined by the equation $4x - 3y + 2z = 20$.

a. Find a vector equation of the line L passing through the points A and B. [3]

b. Find the coordinates of the point of intersection of the line L with the plane Π. [3]

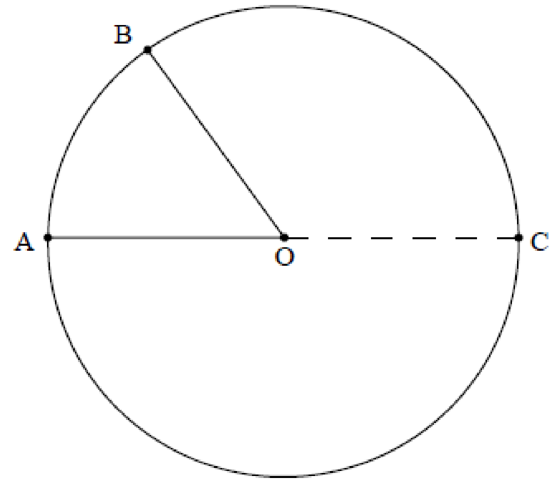
The three vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are given by

$$\boldsymbol{a} = \begin{pmatrix} 2y \\ -3x \\ 2x \end{pmatrix},\ \boldsymbol{b} = \begin{pmatrix} 4x \\ y \\ 3-x \end{pmatrix},\ \boldsymbol{c} = \begin{pmatrix} 4 \\ -7 \\ 6 \end{pmatrix} \text{ where } x, y \in \mathbb{R}.$$

(a) If $\boldsymbol{a} + 2\boldsymbol{b} - \boldsymbol{c} = 0$, find the value of x and of y.

(b) Find the exact value of $|\boldsymbol{a} + 2\boldsymbol{b}|$.

The diagram below shows a circle with centre O. The points A, B, C lie on the circumference of the circle and [AC] is a diameter.



Let $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

a. Write down expressions for $\overrightarrow{AB}$ and $\overrightarrow{CB}$ in terms of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. [2]

b. Hence prove that angle $A\hat{B}C$ is a right angle. [3]

Two planes have equations

$$\Pi_1 : \ 4x + y + z = 8 \text{ and } \Pi_2 : \ 4x + 3y - z = 0$$

Let L be the line of intersection of the two planes.

B is the point on Π_1 with coordinates $(a, \ b, \ 1)$.

The point P lies on L and $A\hat{B}P = 45^\circ$.

a. Find the cosine of the angle between the two planes in the form $\sqrt{\frac{p}{q}}$ where $p, \ q \in \mathbb{Z}$. [4]

b. [6]

(i) Show that L has direction $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

(ii) Show that the point $A \ (1, \ 0, \ 4)$ lies on both planes.

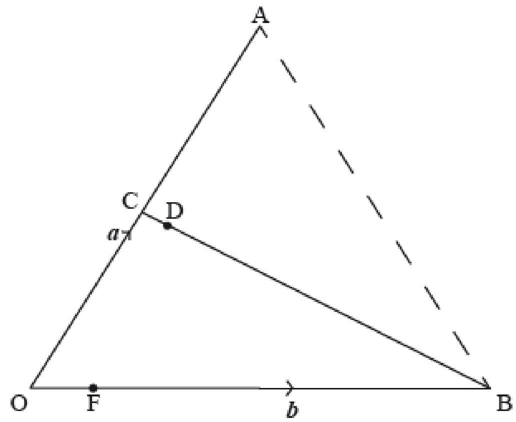
(iii) Write down a vector equation of L.

c. Given the vector $\overrightarrow{AB}$ is perpendicular to L find the value of a and the value of b. [5]

d. Show that $AB = 3\sqrt{2}$. [1]

e. Find the coordinates of the two possible positions of P. [5]

In the following diagram, $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$. C is the midpoint of [OA] and $\overrightarrow{OF} = \frac{1}{6}\overrightarrow{FB}$.



It is given also that $\overrightarrow{AD} = \lambda\overrightarrow{AF}$ and $\overrightarrow{CD} = \mu\overrightarrow{CB}$, where $\lambda,\ \mu \in \mathbb{R}$.

a.i. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ $\overrightarrow{OF}$. [1]

a.ii. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ $\overrightarrow{AF}$. [2]

b.i. Find an expression for $\overrightarrow{OD}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and λ; [2]

b.ii. Find an expression for $\overrightarrow{OD}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and μ. [2]

c. Show that $\mu = \frac{1}{13}$, and find the value of λ. [4]

d. Deduce an expression for $\overrightarrow{CD}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ only. [2]

e. Given that area $\Delta\mathrm{OAB} = k(\text{area } \Delta\mathrm{CAD})$, find the value of k. [5]

The vertices of a triangle ABC have coordinates given by A(−1, 2, 3), B(4, 1, 1) and C(3, −2, 2).

a. (i) Find the lengths of the sides of the triangle. [6]

(ii) Find $\cos \mathrm{B\hat{A}C}$.

b. (i) Show that $\overrightarrow{BC} \times \overrightarrow{CA} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$. [5]

(ii) Hence, show that the area of the triangle ABC is $\frac{1}{2}\sqrt{314}$.

c. Find the Cartesian equation of the plane containing the triangle ABC. [3]

d. Find a vector equation of (AB). [2]

e. The point D on (AB) is such that $\overrightarrow{OD}$ is perpendicular to $\overrightarrow{BC}$ where O is the origin. [5]

(i) Find the coordinates of D.

(ii) Show that D does not lie between A and B.

A line L has equation $\frac{x-2}{p} = \frac{y-q}{2} = z - 1$ where $p,\ q \in \mathbb{R}$.

A plane Π has equation $x + y + 3z = 9$.

Consider the different case where the acute angle between L and Π is θ

where $\theta = \arcsin\left(\frac{1}{\sqrt{11}}\right)$.

a. Show that L is not perpendicular to Π. [3]

b. Given that L lies in the plane Π, find the value of p and the value of q. [4]

c. (i) Show that $p = -2$. [11]

(ii) If L intersects Π at $z = -1$, find the value of q.

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a. For non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that [8]

(i) if $|\boldsymbol{a} - \boldsymbol{b}| = |\boldsymbol{a} + \boldsymbol{b}|$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular;

(ii) $|\boldsymbol{a} \times \boldsymbol{b}|^2 = |\boldsymbol{a}|^2|\boldsymbol{b}|^2 - (\boldsymbol{a} \cdot \boldsymbol{b})^2$.

b. The points A, B and C have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. [7]

(i) Show that the area of triangle ABC is $\frac{1}{2}|\boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{b} \times \boldsymbol{c} + \boldsymbol{c} \times \boldsymbol{a}|$.

(ii) Hence, show that the shortest distance from B to AC is

$$\frac{|\boldsymbol{a} \times \boldsymbol{b} + \boldsymbol{b} \times \boldsymbol{c} + \boldsymbol{c} \times \boldsymbol{a}|}{|\boldsymbol{c} - \boldsymbol{a}|}.$$

Consider the points A(1, −1, 4), B (2, − 2, 5) and O(0, 0, 0).

(a) Calculate the cosine of the angle between $\overrightarrow{OA}$ and $\overrightarrow{AB}$.

(b) Find a vector equation of the line L_1 which passes through A and B.

The line L_2 has equation $\boldsymbol{r} = 2\boldsymbol{i} + 4\boldsymbol{j} + 7\boldsymbol{k} + t(2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k})$, where $t \in \mathbb{R}$.

(c) Show that the lines L_1 and L_2 intersect and find the coordinates of their point of intersection.

(d) Find the Cartesian equation of the plane which contains both the line L_2 and the point A.

ABCD is a parallelogram, where $\overrightarrow{AB} = -\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\overrightarrow{AD} = 4\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$.

a. Find the area of the parallelogram ABCD. [3]

b. By using a suitable scalar product of two vectors, determine whether $\mathrm{A\hat{B}C}$ is acute or obtuse. [4]

The points A, B, C have position vectors $\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $3\boldsymbol{i} + \boldsymbol{k}$ respectively and lie in the plane π .

(a) Find

(i) the area of the triangle ABC;

(ii) the shortest distance from C to the line AB;

(iii) the cartesian equation of the plane π .

The line L passes through the origin and is normal to the plane π , it intersects π at the

point D.

(b) Find

(i) the coordinates of the point D;

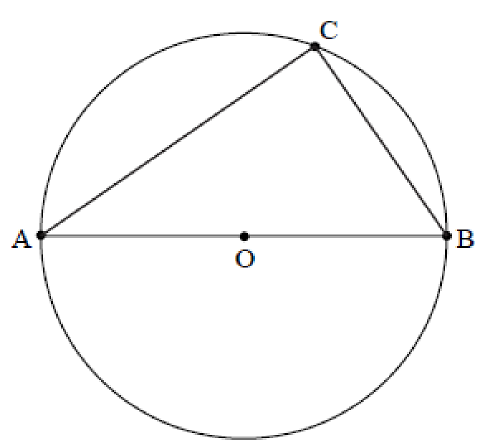
(ii) the distance of π from the origin.

Consider the points A(1, 2, 3), B(1, 0, 5) and C(2, −1, 4).

a. Find $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AC}}$. [4]

b. Hence find the area of the triangle ABC. [2]

In the diagram below, [AB] is a diameter of the circle with centre O. Point C is on the circumference of the circle. Let $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

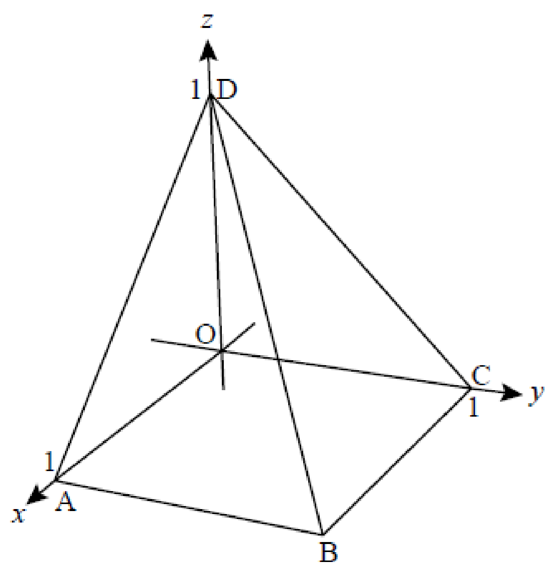


a. Find an expression for $\overrightarrow{\mathrm{CB}}$ and for $\overrightarrow{\mathrm{AC}}$ in terms of $\boldsymbol{b}$ and $\boldsymbol{c}$. [2]

b. Hence prove that $\mathrm{A\hat{C}B}$ is a right angle. [3]

Find the coordinates of the point of intersection of the planes defined by the equations $x + y + z = 3,\ x - y + z = 5$ and $x + y + 2z = 6$.

The following figure shows a square based pyramid with vertices at O(0, 0, 0), A(1, 0, 0), B(1, 1, 0), C(0, 1, 0) and D(0, 0, 1).



The Cartesian equation of the plane Π_2, passing through the points B , C and D , is $y + z = 1$.

The plane Π_3 passes through O and is normal to the line BD.

Π_3 cuts AD and BD at the points P and Q respectively.

a. Find the Cartesian equation of the plane Π_1, passing through the points A , B and D. [3]

b. Find the angle between the faces ABD and BCD. [4]

c. Find the Cartesian equation of Π_3. [3]

d. Show that P is the midpoint of AD. [4]

e. Find the area of the triangle OPQ. [5]

PQRS is a rhombus. Given that $\overrightarrow{PQ} = \boldsymbol{a}$ and $\overrightarrow{QR} = \boldsymbol{b}$,

(a) express the vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$;

(b) hence show that the diagonals in a rhombus intersect at right angles.

The vectors $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$ satisfy the equation $\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$. Show that $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c} = \boldsymbol{c} \times \boldsymbol{a}$.

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a. Consider the vectors $\boldsymbol{a} = 6\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 4\boldsymbol{k}$. [11]

(i) Find the cosine of the angle between vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Find $\boldsymbol{a} \times \boldsymbol{b}$.

(iii) **Hence** find the Cartesian equation of the plane $\prod$ containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ and passing through the point $(1, 1, -1)$.

(iv) The plane $\prod$ intersects the x-y plane in the line l. Find the area of the finite triangular region enclosed by l, the x-axis and the y-axis.

b. Given two vectors $\boldsymbol{p}$ and $\boldsymbol{q}$, [8]

(i) show that $\boldsymbol{p} \cdot \boldsymbol{p} = |\boldsymbol{p}|^2$;

(ii) hence, or otherwise, show that $|\boldsymbol{p} + \boldsymbol{q}|^2 = |\boldsymbol{p}|^2 + 2\boldsymbol{p} \cdot \boldsymbol{q} + |\boldsymbol{q}|^2$;

(iii) deduce that $|\boldsymbol{p} + \boldsymbol{q}| \le |\boldsymbol{p}| + |\boldsymbol{q}|$.

(a) Show that a Cartesian equation of the line, l_1, containing points A$(1, -1, 2)$ and B$(3, 0, 3)$ has the form $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{1}$.

(b) An equation of a second line, l_2, has the form $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{1}$. Show that the lines l_1 and l_2 intersect, and find the coordinates of their point of intersection.

(c) Given that direction vectors of l_1 and l_2 are $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$ respectively, determine $\boldsymbol{d}_1 \times \boldsymbol{d}_2$.

(d) Show that a Cartesian equation of the plane, $\prod$, that contains l_1 and l_2 is $-x - y + 3z = 6$.

(e) Find a vector equation of the line l_3 which is perpendicular to the plane $\prod$ and passes through the point T$(3, 1, -4)$.

(f) (i) Find the point of intersection of the line l_3 and the plane $\prod$.

(ii) Find the coordinates of T$'$, the reflection of the point T in the plane $\prod$.

(iii) Hence find the magnitude of the vector $\overrightarrow{\mathrm{TT}'}$.

The acute angle between the vectors $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ and $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ is denoted by θ.

Find $\cos \theta$.

Let α be the angle between the unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, where $0 \leqslant \alpha \leqslant \pi$.

(a) Express $|\boldsymbol{a} - \boldsymbol{b}|$ and $|\boldsymbol{a} + \boldsymbol{b}|$ in terms of α.

(b) Hence determine the value of $\cos \alpha$ for which $|\boldsymbol{a} + \boldsymbol{b}| = 3\,|\boldsymbol{a} - \boldsymbol{b}|$.

A triangle has vertices A(1, −1, 1), B(1, 1, 0) and C(−1, 1, −1) .

Show that the area of the triangle is $\sqrt{6}$.